CBCS SCHEME

USN | | | | | | | | | | 15MAT41

Fourth Semester B.E. Degree Examination, Feb./Mar. 2022
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Use of statistical tables is permitted.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find y at x = 1.02 correct to four decimal places given $dy = (xy - 1)dx$ and y = 2 at x = 1 by applying Taylor's series method upto third degree term. **(05 Marks)**

b. Use modified Euler's method to compute y(0.1) and y(0.2) given that $\frac{dy}{dx} = x + y$, y(0) = 1 taking h = 0.1. **(05 Marks)**

c. Solve $\frac{dy}{dx} = x - y^2$ with the following data y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762, find y at x = 0.8 by using Milne's predictor-corrector method. **(06 Marks)**

OR

2 a. Use fourth order Runge-Kutta method to find y at x = 0.1 given that $\frac{dy}{dx} = 3e^x + 2y$, y(0) = 0 and h = 0.1. **(05 Marks)**

b. If $\frac{dy}{dx} = 2e^x - y$, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040 and y(0.3) = 2.090, find y(0.4) correct to four decimal places by using Adams-Bashforth method. **(05 Marks)**

c. Use Taylor's series method to obtain a power series in (x – 4) for the equation $5x\frac{dy}{dx} + y^2 - 2 = 0$, $x_0 = 4$, $y_0 = 1$ and use it to find y at x = 4.1 and 4.2 correct to four decimal places. **(06 Marks)**

Module-2

3 a. Given $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$, y(0) = 1, y′(0) = 0. Evaluate y(0.1) using Runge-Kutta method of order 4 **(05 Marks)**

b. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$ **(05 Marks)**

c. Derive Rodrigue's formula

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left[(x^2 - 1)^n\right]$$ **(06 Marks)**

OR

4 a. Obtain the series solution of Bessel's differential equation

$$x^2y'' + xy' + (x^2 - n^2)y = 0$$ **(05 Marks)**

b. Apply Milne's method to solve $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$, given the following table of initial values.

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.399
y′	1	1.2103	1.4427	1.699

Compute y(0.4) numerically. **(05 Marks)**

c. Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre polynomials **(06 Marks)**

Module-3

5 a. State and prove Cauchy-Riemann equations in Cartesian form. **(05 Marks)**

b. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is the circle, $|z| = \frac{3}{2}$ **(05 Marks)**

c. Find the bilinear transformation which maps the points $z = 1, i, -1$ into $w = 0, 1, \infty$. **(06 Marks)**

OR

6 a. If f(z) is analytic function, show that

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] |f(z)|^2 = 4|f'(z)|^2$$ **(05 Marks)**

b. Discuss the transformation $w = z^2$. **(05 Marks)**

c. If $\phi + i\psi$ represents the complex potential of an electrostatic field where $\psi = (x^2 - y^2) + \frac{x}{x^2 + y^2}$, find the complex potential f(z) and hence determine ϕ. **(06 Marks)**

Module-4

7 a. Derive an expression for mean and variance of a Poisson distribution. **(05 Marks)**

b. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) between 65 and 75 [A(1) = 0.3413] **(05 Marks)**

c. The joint probability distribution of two random variables x and y is as follows:

x \ y	- 3	- 2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find (i) Marginal distributions of x and y.
(ii) Covariance of x and y
(iii) Correlation of x and y. **(06 Marks)**

OR

8 a. The number of telephone lines busy at an instant of time is a binomial variate with probability 0.1 that a line is busy. If 10 lines are chosen at random, what is the probability that (i) no line is busy (ii) all lines are busy (iii) atleast one line is busy (iv) atmost 2 lines are busy. **(05 Marks)**

b. The probability that a news reader commits no mistake in reading the news is $\frac{1}{e^3}$. Find the probability that on a particular news broadcast he commits (i) only 2 mistakes (ii) more than 3 mistakes (iii) atmost 3 mistakes. **(05 Marks)**

c. A random variable X has the following probability function for various values of x.

x	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find k (ii) Evaluate $p(x < 6)$, $p(x \geq 6)$ and $p(3 < x \leq 6)$ **(06 Marks)**

Module-5

9 a. Define the terms (i) Null hypothesis (ii) Alternative hypothesis (iii) Critical region (iv) Significance level (v) Confidence limits. **(05 Marks)**

b. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure. 5, 2, 8, −1, 3, 0, 6, −2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure? [$t_{0.5}$ for 11 d.f = 2.201] **(05 Marks)**

c. Find the unique fixed probability vector of the regular stochastic matrix.

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$$

(06 Marks)

OR

10 a. A coin is tossed 1000 times and head turns up 540 times. Decide on the hypothesis that the coin is unbiased. [Given z value at 1% = 2.58] **(05 Marks)**

b. In an exit poll enquiry it was revealed that 600 voters in one locality and 400 voters from an other locality favoured 55% and 48% respectively a particular party to come to power. Test the hypothesis that there is a difference in the locality in respect of the opinion. **(05 Marks)**

c. A gambler's luck follows a pattern. If he wins a game, the probability of winning the next game is 0.6. However if he loses a game, the probability of losing the next game is 0.7. There is an even chance of gambler winning the I game. If so,
(i) What is the probability of he winning the second game?
(ii) What is the probability of he winning the third game?
(iii) In the long run, how often he will win? **(06 Marks)**

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